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INVERSE METHOD OF MEASURING ELECTRICAL CONDUCTIVITY IN A ROTATING MAGNETIC FIELD

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UDC 539.215

Among the noncontact methods available for measuring electrical conductivity [1], an important place is occupied by the method based on the determination of the torque acting on a specimen in a rotating magnetic field [2-5]. The specimen, in the form of a sphere or cylinder, is suspended on a thin elastic filament. The rotating magnetic field is created by means of two- or three-phase current or by the mechanical rotation of coils carrying a direct current. The torque acting on the specimen is determined from the angle of twist of the filament or by compensating for it. In the former case, a light source, mirror, and scale are needed. The electrical conductivity of the specimen is determined by an absolute or relative method. In the first case, use is made of the exact solution of the problem for a conducting sphere or cylinder in a rotating magnetic field [6].

The goal of the present study is to improve the method of measuring electrical conductivity in a rotating magnetic field with the specific aim of checking the purity of metals from their residual resistivity. The procedure for checking purity is based on the fact that at sufficiently low temperatures, the resistivity of metals is determined mainly by impurities and lattice defects. This technique is currently widely used. The purity of a metal is usually characterized by the ratio of its resistivities at room temperature and at the temperature of liquid helium. The main shortcoming of the method which involves the use of a rotating magnetic field is that the specimens must be small – usually about 1 cm. When it is necessary to study the distribution of impurities along long specimens, the only possible means is to cut them into small sections and perform separate measurements for each section. This significantly lengthens the time taken up by the measurement process and increases the consumption of liquid helium. Moreover, this method is more difficult still if the specimen has to be in a sealed ampul for the entire period of measurement.

To overcome these problems, we propose a different method: instead of the torque acting on the specimen, determine the torque acting on the coils which create the magnetic field. A lightweight platform with coils transmitting a two- or three-phase alternating current is suspended on elastic filaments. When the specimen is moved into the space between the coils, eddy currents develop in this space. The interaction of these currents with the rota-

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Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 22-26, July-August, 1989. Original article submitted January 6, 1988; revision submitted March 24, 1988.

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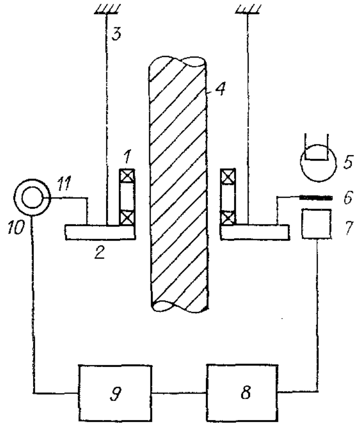

Fig. 1

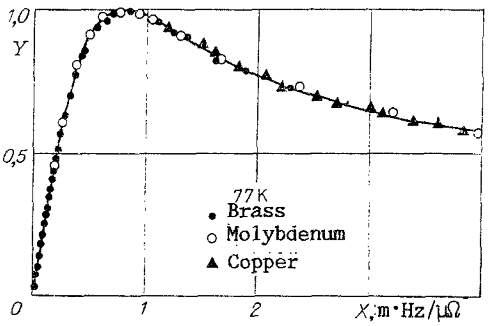

Fig. 2

ting magnetic field creates turning moments which act on both the specimen and the platform with coils. The specimen is set up so that it cannot rotate itself, while the platform with the coils suspended on three thin filaments (trifilar suspension) can rotate. The angle of deflection of the platform is proportional to the acting torque. The size of the section on which electrical conductivity is measured is determined by the length of the region enveloped by the magnetic field, which is, in turn, determined by the diameter of the coils. If the specimen is successively moved further into a system of coils, it is possible to successively determine the conductivity of its different sections. As a result, long specimens can be used for the measurements. It seems natural to refer to the above-described method as the inverse method.

To realize the method, we constructed an appropriate unit (Fig. 1). A system of two pairs of coils 1 with mutually perpendicular axes (one pair of coils is shown) is secured to a lightweight platform 2 with a hole in its center. The platform is suspended on three thin filaments 3 (two of which are shown). The cylindrical specimen 4 can be moved along the vertical axis inside the system of coils. The rotating magnetic field is created with the passage of a two-phase alternating current with a 90° phase shift through the coils (the current is supplied by a two-phase generator). The frequency of rotation of the magnetic field can be set at values from 0.001 to 2000 Hz, which makes it possible to measure electrical resistivity in the range 10^{-13}-10^{-6} Ω·m. The turning moment acting on the platform with the coils is determined by the compensation method. Here, we use a sensor to detect the rotation of the platform. The sensor consists of a light source 5, a flag indicator 6 rigidly connected to the platform, and a photodetector 7. The output voltage of the photodetector is sent to the input of a dc amplifier 8. The output current of the amplifier, recorded by digital milliammeter 9, enters the stationary solenoid 10 interacting with solenoid 11 secured to the platform and supplied with dc current. The interaction of the two solenoids creates a compensating moment which is nearly equal to the turning moment acting on the system of coils. This near equality is achieved as a result of the high gain of the compensation circuit. Thus, during the time of measurement, the platform with coils is barely displaced from its initial position. Meanwhile, the current in solenoid 10 is proportional to the turning moment which develops due to the interaction of the induction currents in the specimen with the currents in the coils.

Electrical conductivity was calculated as follows. The length of the specimens was considerably greater than the length of the magnetic field, so that the specimens could be considered infinitely long. The turning moment per unit length of the specimen [6]

$$M = \frac{rH^2}{|k|^2}\,\mathrm{Re}\left[\frac{kJ_1(kr)}{J_0(kr)}\right]. \tag{1}$$

Here, $k^2 = 2\pi i f \mu_0/\rho$; H is the strength of the magnetic field; r is the radius of the specimen; J_0 and J_1 are zero- and first-order Bessel functions; f is the frequency of rotation of the magnetic field; ρ is the electrical resistivity of the specimen; μ_0 is the magnetic constant (all of these quantities are expressed in SI units).

It is not hard to show that the maximum turning moment is proportional to the square of the current in the coils and the square of the specimen radius and is independent of its

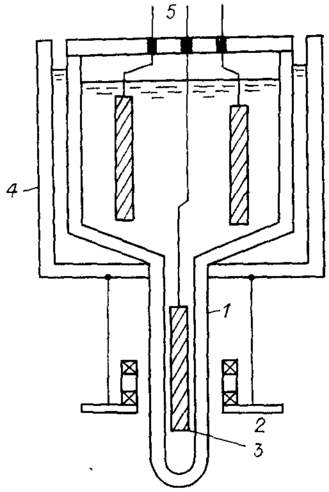

Fig. 3

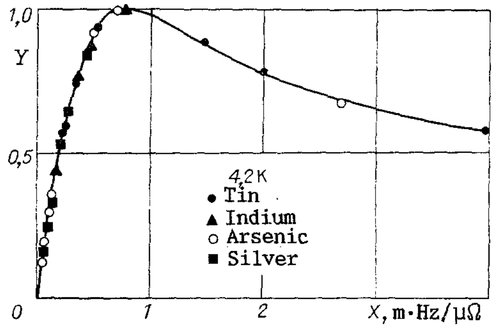

Fig. 4

electrical conductivity. Thus, for each unit, it is possible to find the dependence of the maximum torque acting on the specimen (in our case, on the platform with coils) on the current in the coils and the radius of the specimen.

It also follows from Eq. (1) that the ratio of the torque M to its maximum value M_0 is a universal function of $X = r^2f/\rho$. Having measured the specimen radius and the torque acting on the platform with coils, we find $Y = M/M_0$ and we then use the theoretical relation Y(X) to determine resistivity. We only need to know the region of values of X in which the measurements were made — above or below the torque maximum; this is easily determined experimentally. In the present case, we can use not only the initial linear part of the relation Y(X), but also the nonlinear part. We can even use the region of values after the maximum.

To check the validity of the proposed inverse method, we measured the electrical conductivity of certain metals at room temperature and the boiling points of nitrogen and helium (77 and 4.2 K). We used specimens of copper, molybdenum, and brass 16.5 mm in diameter at room and nitrogen temperatures. We used these materials to prepare specimens for contact measurements so that we could either directly compare the data obtained by both methods or calculate X from the results of the contact measurements. When the second method was used, we calculated resistivity on the basis of contact measurements for each measurement made by the inverse method X. The experimental values of Y were compared with the theoretical values (line in Fig. 2).

We used a special cryostat (Fig. 3) for the measurements at helium temperatures. The narrow part of a helium Dewar flask 1 was passed through the platform with coils 2, while the specimen 3 was moved in the flask along the vertical axis by means of a rod. The platform with coils was suspended inside another Dewar flask with liquid nitrogen 4. Three specimens were placed simultaneously in the wide part of the helium flask. All of the measurements were made without opening the helium flask (the specimens were replaced by turning and lowering the rods with the specimens 5). The earth's magnetic field was compensated for by means of Helmholtz coils.

At the helium temperatures, we used specimens of tin, indium, arsenic, and silver from 13 to 18 mm in diameter. The contact measurements could not be made in this case, so we proceeded as follows. We found electrical resistivity by the inverse method for each specimen, using different frequencies of rotation of the magnetic field. We then determined the mean value, which was used to calculate X. We were then able to compare experimental values of Y with theoretical values (line in Fig. 4). The results fully confirmed the validity of the inverse method of measurement within a broad range of electrical resistivities.

The theoretical relation Y(X) was tabulated with an error no smaller than 0.1% [7]. The radius of the specimens was determined with an error of 1%, the current in the coils was determined with an error no greater than 0.5%, and the frequency of the current was determined with an error smaller than 0.1%. The greatest error was introduced by the measurement of the torque — about 2%. The total error of the determination of electrical conductivity was roughly 3%. Thus, the error is nearly the same as that of the conventional method involving the use of a rotating magnetic field.

Consequently, the inverse method eliminates the main shortcoming of the existing method, making it possible to use specimens of almost any length and to make local measurements of conductivity. This in turn makes it easier to perform measurements within a broad temperature range and to shorten measurement time. Automation of the measurement process is also made easier.

Comparing the method of a rotating magnetic field with other noncontact methods, we should note one important advantage of the former. When use is made of the method of eddy current decay or the method of the effective magnetic susceptibility of specimens in an alternating magnetic field, the recorded signal decreases with an increase in electrical conductivity. This takes place because of an increase in the time of decay of the eddy currents (in the eddy current method) and because magnetic susceptibility has to be measured at lower magnetic-field frequencies (in the susceptibility method). Measurement of the electrical conductivity of pure metals at helium temperatures requires recording equipment with a sensitivity on the order of 0.1 μV. Even greater sensitivity is required with a further increase in the degree of purity of the metals. In contrast, in the method of the rotating magnetic field, the maximum torque is independent of electrical conductivity and the recording system is universal. This advantage may prove to be decisive in certain cases.

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